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## Vertical Directionality of a Source in Shallow Water

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### Abstract

The vertical directionalities of the ambient noise and a distance source are usually not same. Utilizing their difference, one can steer the array carefully to improve the output signal to noise ratio (SNR) and increase the passive detection range against a submerged target in shallow water. Due to the multipath effect, the signal will not always arrive from the horizontal direction. The directional response of the vertical line array (VLA) to a distance source can be expressed in terms of the modal beams weighted in accordance to the normal mode amplitude. This modal representation offers the physical interpretation of the vertical directionality of the source in terms of normal modes. When the short VLA lies at the lower sound speed water column and the high frequency source locates at the larger sound speed water column, there always exists a notch in the horizontal direction. The vertical directionality of the source has been validated using the Mediterranean data.

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*Keywords:* array response; vertical line array; shallow water

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### 1. Introduction

The vertical directionality of the noise is often relatively stable in a short period of time for a given region whether the noise notch is present[1-3]. When the noise notch is present, it creates a window to look through the signal arriving in the horizontal direction. However, due to the multipath effect on sound propagation in the waveguide[4], the signal is often split into several beams with different elevation angles. Thus, the signal does not always arrive from the horizontal direction

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The directional response of the array to distance source is primarily controlled by the physics of sound propagation in the waveguide. In this study we present a modal analysis of the arrival structure of distance source in shallow water. Normal mode theory expresses the acoustic pressure field in terms of a normal mode expansion. The directional response of the VLA to distance source in the waveguide is expressed in terms of the modal beams weighted in accordance to the normal mode amplitude. In fact, a normal mode is a superposition of up and down going plane waves of equal amplitude and vertical wavenumber  $k_{zm}$ . Both of these waves are propagating at grazing angles  $\theta_m = \arctan(k_{zm}/k_{rm})$ , where  $k_{rm}$  is the horizontal wavenumber. The low and high order modes arrive at shallow and steep grazing angles, respectively.

These modal representations offer the physical interpretation of the angular distribution of the array response in terms of normal modes. In this study, we will explain the physical mechanism of angular deviation of the array response to distance source in shallow water. Further, the existence of the notch of the VLA in horizontal direction has been confirmed experimentally using the Mediterranean data[5-7].

## 2. Theoretical Model

Normal mode method is convenient because it is accurate and computationally efficient for low frequency and far field. According to the normal mode theory, acoustic pressure field can be expressed in terms of a normal mode expansion. The eigenfunctions and eigenvalues are solutions to the Helmholtz equation under the boundary conditions. The far-field acoustic pressure field is then the weighted sum of the contributions from each mode[9]

$$p(r, z; r_s, z_s) = \frac{i}{\rho(z_s)\sqrt{8\pi(r-r_s)}} e^{-i\pi/4} \sum_{m=1}^M \psi_m(z_s) \psi_m(z) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \quad (1)$$

where eigenfunction  $\psi_m$  describes the mode shape for mode  $m$ , and  $k_{rm}$  is corresponding horizontal wavenumber.  $M$  is the number of contribution modes.  $(r_s, z_s)$  is the source position in cylindrical coordinates, and  $(r, z)$  is the receiver position. For an isotropic point source, the harmonic time dependence of  $e^{-i\omega t}$  can be neglected. Thus, the data vector observed on the VLA at range  $r$  is

$$x(\theta_0) = [p(r, z_1; r_s, z_s), p(r, z_2; r_s, z_s), \dots, p(r, z_N; r_s, z_s)]^T \quad (2)$$

For the simplicity, the array beam  $y(\theta)$  is chosen to discuss instead of the beam power  $Y(\theta)$ . one obtains

$$y(\theta) = \frac{i}{\rho(z_s)\sqrt{8\pi(r-r_s)}} e^{-i\pi/4} \times \mathbf{w}(\theta)^T \begin{bmatrix} \sum_{m=1}^M \psi_m(z_s) \psi_m(z_1) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \\ \sum_{m=1}^M \psi_m(z_s) \psi_m(z_2) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \\ \vdots \\ \sum_{m=1}^M \psi_m(z_s) \psi_m(z_N) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \end{bmatrix} \quad (3)$$

Using the vector notation

$$\mathbf{\Psi}_m = [\psi_m(z_1), \psi_m(z_2), \dots, \psi_m(z_N)]^T \quad (4)$$

Equation (3) becomes

$$y(\theta) = \frac{i}{\rho(z_s)\sqrt{8\pi(r-r_s)}} e^{-i\pi/4} \times \sum_{m=1}^M \psi_m(z_s) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \mathbf{w}(\theta)^T \mathbf{\Psi}_m \quad (5)$$

The vector  $\mathbf{\Psi}_m$  is the spatial sampling of the  $m$ th mode at each hydrophone. The normal mode amplitudes 18 generated by the source are given by

$$A_m = \frac{ie^{-i\pi/4}}{\rho(z_s)\sqrt{8\pi(r-r_s)}} \psi_m(z_s) \frac{e^{ik_{rm}(r-r_s)}}{\sqrt{k_{rm}}} \quad (6)$$

The modal beam  $B_m$  is the array response of the VLA to the  $m$ th mode,

$$B_m = \mathbf{w}(\theta)^T \mathbf{\Psi}_m \quad (7)$$

Then Eq. (5) can be written as

$$y(\theta) = \sum_{m=1}^M A_m B_m \quad (8)$$

The array beam of the VLA to distance source in the waveguide can be decomposed into modal beams. The contribution of each modal beam depends on the normal mode amplitude  $A_m$  in accordance to the source depth.

### 3. Experimental data analysis

#### 3.1. Experiment description

The existence of the notch of the VLA response in the horizontal direction is also confirmed using the vertical array data collected during the October 1993 Mediterranean Sea trial[5-7]. This area is characterized by a flat bottom covered with clay and sand-clay sediments. The propagation conditions were typical downward refracting summer conditions. The vertical array contained 48 hydrophones at 2 m spacing with a total aperture of 94 m. The bottom hydrophone was at a depth of 112.7 m and the top hydrophone was correspondingly at a depth of 18.7 m. A detailed description of the experimental dataset may be found in Ref. 5-7.

#### 3.2. Signal data

A stationary source was deployed at a depth of 80 m and approximately 5.6 km. One of signals transmitted by the stationary source was a continuous transmission of pseudorandom noise (PRN) produced using a maximal length sequence. The center frequency was 335 Hz, and the  $-3$  dB bandwidth was approximately 30 Hz. It should be noted that the experimental data used here contained signal and noise. The input SNR was high, about 10 dB. Since the narrow-band problem was of interest here, the narrow-band time-domain beamforming is used to analysis the signal vertical directionality. The signal data are filtered through a FIR bandpass filter between 333 and 337 Hz. 60 seconds of data were averaged to form the covariance matrix  $\mathbf{R}_x(\theta_0)$ .

Figure 1 is a typical result showing the signal beam pattern as a function of array depth and steering angle. The VLA consists of 21 continuous hydrophones. The array depth is the depth of the middle

hydrophone of the subarray. Attention is restricted to  $\pm 40^\circ$  of horizontal. The signal notch at  $0^\circ$  is clearly visible over all the subarray at different depths. The maximum of the response of the VLA is away from the horizontal direction and the signal is evident on the up- and down-looking beams.

Figure 2 shows the FFT analyzed power spectra of the sub-array output using the data collected during the Mediterranean Sea trial. The sub-array contain the top 21 hydrophones (18.7~58.7 m). It can be seen that in Figure 2(a) the signal spectral components (320~350 Hz) are just noticeable when the array steers to the horizontal direction. The corresponding average output SNR is about 13.6 dB. However, when the array steers to the strongest signal power direction, the signal is significantly above the noise background as shown in Figure 2(b), and the average output SNR is about 23.7 dB.

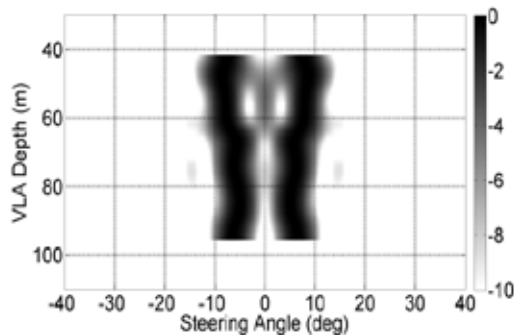


Figure 1. Signal beam pattern observed by the sub-VLA consists of 21 continuous hydrophones.

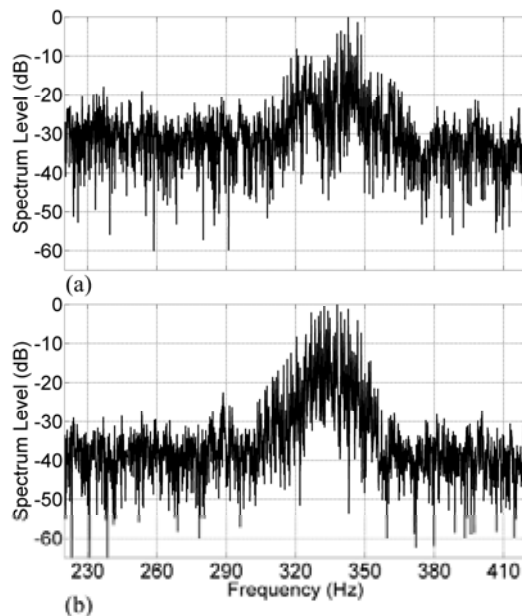


Figure 2. Normalized output power spectra of the sub-array: top 21 hydrophones (18.7~58.7 m). (a) array steering angle  $\theta = 0^\circ$ , (b) array steering angle  $\theta = 7.6^\circ$  which corresponds to the strongest signal power direction.

#### 4. Conclusions

Due to the signal is often split into several beams with different elevation angles in shallow water, the DOA mismatch on conventional plane-wave beamformers will significantly degrade the array gain. In this study, the directional response of the short VLA to distance source in shallow water is investigated through normal mode modeling and computer simulation.

For the summer environment studied, the low order modes are confined to the lower water column. When the short VLA locates at the lower water column, there is always a deep notch of the array response at the near horizontal direction or sources at the upper water column. It was also tested for a variety of different conditions, such as different bottom types, and different SSPs. Based on the above results, three conditions of the existence of the notch at high (e.g., 1500 Hz) frequencies are obtained. 1) SSP should have a gradient; 2) The short VLA lies at the lower sound speed water column where the low order modes are confined to; 3) The distance source locates at the larger sound speed water column where it coupled weakly with low order modes and strongly with the higher modes.

Finally, the vertical directionality of the signal with short VLAs has been analyzed and discussed using the Mediterranean data. One can steer the array carefully to improve the output SNR and increase the passive detection range against a submerged target in shallow water.

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